

# Moving Quark in a Viscous Fluid

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## Abstract

To study the rate of energy and momentum loss of a heavy quark in QGP, specifically in the hydrodynamic regime, we use fluid/gravity duality and construct a perturbative procedure to find the string solution in gravity side. We show that by this construction, the position of the world-sheet horizon and thereby the drag force exerted on the quark can be computed perturbatively, order by order in a boundary derivative expansion, for a wide range of quark velocities. At ideal order, our result is just the localized expression of the drag force exerted on a moving quark in thermal plasma, while for a quark whose velocity does not belong to the mentioned range, we predict a nonlocal drag force. Furthermore, we apply this procedure to a transverse quark in Bjorken flow and compute the first-derivative corrections, namely viscous corrections, to the drag force. We show that the diffusion time depends on the initial momentum of the transverse quark and can be about 4fm for a high energy quark which has been created sufficiently late.

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## 1 Introduction

Studying systems out of equilibrium is an interesting subject in physics. Depending on how it evolves towards equilibrium, there are different methods to study the system. In a familiar case, when the deviation from equilibrium has small amplitude everywhere, one can apply linear response theory to study the dynamics of the system. Computing retarded Green's functions in equilibrium, we can extract certain information about the dynamics of the system slightly deviated from its equilibrium state. However, when the understudying

system is described by a strongly coupled field theory, we are not able to do such computations through perturbative methods. In such cases, AdS/CFT correspondence appears as a powerful tool.

AdS/CFT conjecture in its original statement, relates the type IIB string theory on  $\text{AdS}_5 \times S^5$  space-time to the four-dimensional  $\mathcal{N} = 4$  SYM gauge theory [1]. On the other hand, there are many examples which have been studied with the holographic description of AdS/CFT in which, a strongly coupled field theory living on the boundary of the AdS space is pictured to the weakly coupled gravity theory in the bulk of AdS [2]. Accordingly, this holographic picture, provides a tool to study the strongly coupled systems out of equilibrium [3].

Although our familiar strongly coupled systems do not exactly coincide with the  $\mathcal{N} = 4$  SYM gauge theory, we apply this theory as a useful model to study such systems through AdS/CFT correspondence. This modelling shows agreement with real physical systems in particular circumstances. For example in QGP experiments at RHIC or at LHC, a gas of quarks and gluons is produced with the temperature about 170 MeV which is strongly coupled. Although QCD as the underlying theory in these experiment is distinct from  $\mathcal{N} = 4$  SYM gauge theory, but this has not prevented us to apply this to study of QCD in QGP experiments. Specially in the hydrodynamic regime, computation of the transport coefficients is known as a remarkable success of AdS/CFT [4] providing results in agreement with experiment.

One of the other significant subjects from both experimental and theoretical stand-points, is the motion of quarks created during heavy-ion collisions in the plasma. Up to now, invoking AdS/CFT, three different mechanisms have been introduced to describe the rate of energy and momentum loss through this motion; exerting drag force on the quark from the plasma [5, 6], Cherenkov radiation of a fast heavy quark [7] and radiation related to quark's deceleration [8, 9]. There is a simple holographic picture for these problems. The probe quark moving in plasma is mapped to a probe string in the AdS space. So instead of taking into account the quark motion in a strongly coupled system one can simply trace the classical dynamics of a string in the gravity side. We will concentrate on the first case in this paper. A common part in both related seminal works [5, 6] is that the authors have studied a the uniform motion of a quark in the boundary plasma. This quark is the end point of a string stretching in the AdS bulk. So in order to move uniformly, it should be forced by an external source, i.e. an electric field. The momentum rate flowing down to the string has been interpreted as drag force exerted from the plasma on the quark. It has been shown in [5, 6] that when a quark is dragged with a constant velocity  $v$  in a thermal plasma at temperature  $T$ , the rate of the energy and momentum loss of the quark in the

plasma frame are given by

$$\frac{dp}{dt} = \frac{1}{v} \frac{dE}{dt} = -\frac{\pi}{2\alpha'} T^2 \frac{v}{\sqrt{1-v^2}}. \quad (1.1)$$

In these derivations, based on AdS/CFT correspondence, the gravity set up dual to the thermal field theory has been taken as a uniform 5-dimensional AdS black brane with the metric

$$ds^2 = G_{MN} dx^M dx^N = \frac{dr^2}{r^2 f(br)} + r^2 (P_{\mu\nu} - f(br) u_\mu u_\nu) dx^\mu dx^\nu \quad (1.2)$$

where  $M = (r, \mu)$ ,  $f(r) = 1 - \frac{1}{r^4}$  and the temperature of the black brane is given by  $T = 1/\pi b$ . Also  $P_{\mu\nu} = u_\mu u_\nu + \eta_{\mu\nu}$  is a projection tensor where  $u^\mu$  is the four-velocity of the plasma. For a plasma at rest in the laboratory frame  $u^\mu = (1, 0, 0, 0)$ .

From the experimental viewpoint, an important case to study is a moving quark in an expanding plasma where, local thermal equilibrium has been attained. It can be generated in the time interval between thermalization and hadronization, after scattering of two massive nuclei [10]. In this time interval, the excited matter is expanding and cooling down such that at any point, hydrodynamic description is valid and therefore, dynamics of the expansion can be described by the corresponding equations of motion [11]. Beforehand AdS/CFT, there was not any theoretical approach to study the hydrodynamic regime in strongly coupled field theories. In fact, AdS/CFT has opened a new window to research: study of CFT hydrodynamic from gravity. First efforts in this context were made by [12, 13] which were confirmation of existence of the hydrodynamic regime in a  $\mathcal{N} = 4$  SYM gauge theory. In another remarkable development, Janik and Peschanski found an asymptotic gravity dual to the hydrodynamical flow in QGP, namely Bjorken flow, from a nonlinear derivation [14]. In 2007, BHMNR introduced the full version of fluid/gravity duality [15] in which, they showed that there is a one to one map between fluid dynamical flows of a CFT on the boundary with the long wavelength perturbations of the AdS black brane in the bulk.

It is natural to investigate the quark motion in a dynamical flow as a phenomenological problem related to the QGP experiments. In this direction, the drag force exerted on a quark moving in Bjorken flow has been computed in [16] and later in [17]. In these works, Bjorken fluid has been considered as an ideal fluid. To be more precise, one can go further in the hydrodynamic expansion and study the viscous effects. For example, by applying the gravity set up found in [18], it is possible to compute the drag force exerted on a quark moving in viscous Bjorken fluid.

As discussed, fluid/gravity correspondence maps each long wavelength perturbation around an AdS black brane to a hydrodynamical flow on the boundary of AdS. Apart from sketching the mentioned map, a key point in fluid/gravity duality is that it provides

a perturbative technology to find the regular solutions in AdS space. Similarly, based on fluid/gravity duality, we construct a perturbative procedure to find the classical string solution in the long wavelength perturbed AdS background. The choice of coordinates has a key role in constructing our perturbative procedure. We exploit the Eddington-Finkelstein coordinates in which the extension of the string in the boundary directions is finite. It is in contrast to the coordinates systems taken in [5, 6] where, the embedding of the string has an infinite extension in the boundary spatial directions. This delicate point is the origin of our perturbative procedure.

It is important to note that constructing of such perturbative procedure is the key issue in development of our idea in this work. For this procedure to be applicable, it is necessary that the relative velocity of fluid and quark to be lower than a relativistic upper-bound velocity which turns out to be about 0.97. Using this procedure, one can compute certain physical quantities related to a quark in fluid medium perturbatively. For example we proceed to compute the drag force exerted on a quark moving in a fluid dynamical flow. As a main result, we find that at ideal order, it can be computed in a flow-independent manner. In addition, we extend this procedure to higher orders of perturbation. We show that in a wide range of the quark velocities, one can compute corrections to the drag force perturbatively, order by order in a boundary derivative expansion. Finally, we apply this method to a transverse quark created in Bjorken fluid and compute the drag force perturbatively up to viscous order. Our numerical computing shows that the diffusion time depends on the creation time of the quark, with respect to the collision time. For a transverse quark created at  $\tau = 15 fm$ , we estimate it about  $4 fm$ . This is an improvement of the result of [19] where compared to the result of [20]. We have considered a varying temperature profile through the motion which was lacking in previous approaches.

This paper is organized as follows. We begin the section 2 with a brief review of drag force derivation in String/CFT duality in a world-sheet covariant approach. In § 3 we apply the statement of § 2 to compute the space-time covariant drag force exerted on a quark in a global boosted plasma in. § 4 is devoted to the core of this paper in which after reviewing some key points in fluid/gravity correspondence, we explain our perturbative procedure. Then, we apply this method to find classical string solution in gravity background dual to a boundary fluid dynamical flow. As a preliminary application of our perturbation method, we study a quark moving in a slowly time varying plasma in § 5. In § 6 we apply our perturbation to Bjorken fluid and compute first-derivative correction to the drag force exerted on a transverse quark in this fluid. In particular, we find the diffusion time of the quark. Finally in § 7 we end with a discussion of results and of possible future projects.

## 2 World-sheet covariant description of drag force

Consider a quark moving with velocity  $u_q$  in the boundary field theory. According to the string/CFT duality, such quark is the end point of an open string, embedded in a one higher dimensional space-time dual to the boundary field theory. The dynamics of the classical string is described by the Nambu-Goto action as follows

$$S = -\frac{1}{2\pi\alpha'} \int d\sigma d\tau e^{\frac{\Phi}{2}} \sqrt{-g}, \quad \sigma \in [0, \sigma_1] \quad (2.1)$$

where  $\Phi$  is the background dilaton field. Also in the above expression,  $g = \det g_{\alpha\beta}$  where  $g_{\alpha\beta} = G_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu$ . Free motion of the quark in the plasma is dissipative. AdS/CFT duality ascribes this dissipation effect to flowing the momentum down to the string in the bulk. Therefore, to have a uniform motion, the quark should be forced by an external source such as an electric field on the boundary. In such a case, the mentioned flow rate is interpreted as the drag force exerted on the quark.

By construction, drag force should be a vector in space-time and a scalar on the world-sheet. In order to have it in a world-sheet covariant form, consider the world-sheet currents as following

$$\Pi_M^\alpha = \frac{\partial \mathcal{L}}{\partial(\partial_\alpha X^M)} = -\frac{e^{\frac{1}{2}\Phi(x^\mu)}}{2\pi\alpha'} \sqrt{-g} P_M^\alpha \quad (2.2)$$

where  $\alpha$  and  $M = r, \mu$  describe the world-sheet and space-time directions respectively. In contrast to  $P_M^\alpha$ , these currents are not the world-sheet vectors. According to the previous paragraph discussion, to compute drag force we should find ingoing flux of the energy and momentum across the time-like boundary of the world-sheet, the  $\sigma = \sigma_1$  curve. Assuming  $n^\alpha$  as the inward normal vector to this curve, the covariant 4-drag force is given by:

$$F_\mu = -P_\mu^\alpha n_\alpha. \quad (2.3)$$

It is clear that the choice of the gauge determines what combination of  $P_M^\alpha$ s is equal to the drag force. Let us recall that the relativistic four-force,  $F^\mu = \frac{d}{d\tau} p^\mu$ , exerted on the quark should be perpendicular to the quark four-velocity:

$$F_\mu u_q^\mu = 0. \quad (2.4)$$

For a quark moving with velocity  $u_q$  in the 1-direction of the plasma frame, applying (2.4) to  $F_\mu = \gamma (\frac{dE}{dt}, \frac{dp}{dt}, 0, 0)$  ensures that

$$\frac{dp}{dt} = \frac{1}{u_q} \frac{dE}{dt} \quad (2.5)$$

which can be obviously observed in (1.1) and all of our perturbative results in next sections.

### 3 Moving quark in a boosted thermal plasma

Transforming the coordinates in AdS black brane metric (1.2) to Eddington-Finkelstein coordinates, we find the metric of gravity space-time dual to the boundary boosted thermal plasma:

$$ds^2 = G_{MN}dx^M dx^N = -2u_\mu dx^\mu dr + r^2 (P_{\mu\nu} - f(br)u_\mu u_\nu) dx^\mu dx^\nu \quad (3.1)$$

where, the boosted plasma with velocity  $u^\mu$  and temperature  $T = \frac{1}{\pi b}$  lives on the boundary of space-time,  $r \rightarrow \infty$ . It is significant to note that (3.1) introduces an exact 4 + 1 parameters family of Einstein equations solutions [21].

To study the energy and momentum loss of a moving quark in this system, we consider the four-velocity of plasma to be  $u_f^\mu = (u^0, u^1, 0, 0)$  and for simplicity assume that the quark is moving in the 1-direction with a constant velocity  $u_q$ . Above-mentioned quark is the end point of an open string embedded in a 5-dimensional space-time by the metric (3.1). Dynamics of the string is governed by the Nambu-Goto action (2.1) where dilaton factor can be dropped in the above background. In the static gauge  $(\sigma, \tau) = (r, t)$ , embedding of the string is given by  $X^M(r, t) = (r, t, X^i(r, t))$ . Plugging

$$X^1 = u_q t + \xi(r), \quad X^2 = X^3 = 0 \quad (3.2)$$

in (2.1), we reach to following expression:

$$\mathcal{L} = \sqrt{A\xi'^2 + 2B\xi' + C} \quad (3.3)$$

where  $A$ ,  $B$  and  $C$  are some constant coefficients which depend on the bulk metric elements and the velocity of the quark (A.1). As it can be observed,  $\xi$  enters into the world-sheet lagrangian only through its derivative  $\xi'$ . So instead of solving EOM, one can exploit the conservation equation of the conjugate momentum of  $\xi$ :  $\pi_\xi = \partial\mathcal{L}/\partial\xi'$ . Deriving  $\xi'$  in terms of  $\pi_\xi$ , we find

$$\xi' = -\frac{B}{A} \pm \frac{1}{A} \sqrt{\pi_\xi^2 \frac{B^2 - AC}{\pi_\xi^2 - A}}. \quad (3.4)$$

It can be simply checked that  $B^2 - AC$  vanishes at some point in the bulk. Obviously, for the second term to be real and so the induced metric to be non-degenerate, numerator and denominator should have common root, namely  $r^*$ . Solving  $B^2 - AC = 0$ ,  $r^*$  is obtained

as

$$r^* = \frac{1}{b} \sqrt{\frac{1 - \beta u_q}{\sqrt{1 - \beta^2} \sqrt{1 - u_q^2}}}. \quad (3.5)$$

Demanding the denominator to vanish at  $r^*$ , determines the value of  $\pi_\xi$  as:

$$\pi_\xi = \sqrt{A(r^*)} = \frac{1}{b^2} \frac{u_q - \beta}{\sqrt{1 - u_q^2} \sqrt{1 - \beta^2}} \quad (3.6)$$

where  $\beta = \frac{u^1}{\sqrt{1 + u^{12}}}$  is the 1-component of the plasma velocity in the boundary lab frame (LF). Inserting (3.6) in (3.4), we have:

$$\xi' = \frac{u_q - \beta}{r^2 f(br) \sqrt{1 - \beta^2}} (-1 \pm \frac{1}{b^2 r^2}), \quad (3.7)$$

where the choice of  $+$  or  $-$  depends on the sign of  $u_q - \beta$ ; for  $u_q > \beta$  ( $u_q < \beta$ ), the  $+$  ( $-$ ) is acceptable.

### 3.1 Drag force computation

Let us recall from our discussion in § 2 that the above gauge choice, fixes the inward normal vector to the boundary of the world-sheet at  $r \rightarrow \infty$  as  $n_\alpha = (-1, 0)$ . So the 1-component of 4-drag force may be written as:

$$F^1 = F_1 = P_1^r = \left( \frac{1}{\sqrt{-g}} \Pi_1^r \right)_{r \rightarrow \infty} \quad (3.8)$$

where

$$\Pi_1^r = -\frac{1}{2\pi\alpha'} G_{1\nu} \frac{(\dot{X} \cdot X)(\dot{X}^\nu)' - (\dot{X})^2 (X^\nu)'}{\sqrt{-g}} = -\frac{1}{2\pi\alpha'} \pi_\xi. \quad (3.9)$$

Translational invariance of (3.3) ensures the conservation of the world-sheet currents,  $\partial_\alpha \Pi_\mu^\alpha = 0$ . Knowing the value of  $\Pi_1^r$  at  $r^*$ , one can integrate this equation to compute  $\Pi_1^r$  as a function of radial coordinate. Since the space-time is static, it turns out that  $\Pi_1^r$  is constant along the string and equal to  $\Pi_1^r(r^*)$ . Therefore we obtain the 1-component of 4-drag,  $F^\mu = (\gamma \frac{dE}{dt}, \gamma \frac{dp}{dt}, 0, 0)$ , as follows:

$$F^1 = \frac{1}{\sqrt{1 - u_q^2}} \frac{dp}{dt} = -\frac{1}{2\pi\alpha'} \frac{1}{\sqrt{1 - u_q^2}} \pi_\xi \quad (3.10)$$

where  $dp/dt$  is the 1-component of 3-drag force in LF.

It can be simply seen that for  $u_q > \beta$ , drag force is pointed to opposite direction of the



quark motion while for  $u_q < \beta$  aligns in the quark direction of motion. Expectedly, when the quark is moving with the velocity equal to that of the plasma, does not sense any drag force.

To compute the rate of energy loss, we should work out  $\Pi_1^t$ . After a simple calculation we have

$$F^t = \frac{1}{\sqrt{1-u_q^2}} \frac{dE}{dt} = -\frac{1}{2\pi\alpha'} \frac{u_q}{\sqrt{1-u_q^2}} \pi\xi. \quad (3.11)$$

It is interesting to note that for  $\beta = 0$ , the rate of energy and momentum loss in the LF obtained from (3.10) and (3.11) exactly coincide with (1.1). It is expectable that one could obtain the above expression by a lorentz transformation of (1.1). In addition, (3.10) and (3.11) remind us that instead of studying string picture dual to a moving quark in global thermal plasma, one can apply string/CFT duality to a rest quark in a global boosted thermal plasma, since both  $(u_q = v, \beta = 0)$  and  $(u_q = 0, \beta = -v)$  cases have common results from (3.6).

### 3.2 Embedding of the string

The embedding of the string in the bulk space-time is the solution of equation (3.7). Assuming  $u_q > \beta$  and picking up the plus sign to ensure the momentum flowing down the string, we have

$$\xi(r) = -\frac{u_q - \beta}{\sqrt{1 - \beta^2}} b \left( \tan^{-1}(rb) - \frac{\pi}{2} \right) \quad (3.12)$$

where we have chosen the constant of integration such that  $X^1$  to be identified by  $x = vt$  on the boundary. In Fig.1 the embedding of the string has been compared in two different coordinates systems. Figure (a) shows the profile of the string for  $u_q > \beta$  in a constant time slice of the metric (1.2). In case (b) we have shown the same string has been considered in the coordinates system given in (3.1). Although the quark is moving to the right, the string trails out in front of the quark in this case. It is not so surprising because, the time coordinate in (3.1) is the ingoing Eddington-Finkelstein time in which all of events on an ingoing null geodesic are simultaneous.

In contrast to the former case in which the extension of the string in the 1-direction is infinite, in the latter case, the string has occupied a finite range in the 1-direction. Substituting the value of the horizon radius in (3.12) gives the length of this finite extension in the LF as following:

$$\Delta x^1 = \frac{\pi}{4} b \frac{u_q - \beta}{\sqrt{1 - \beta^2}}. \quad (3.13)$$

The finiteness of  $\Delta x^1$ , as a direct consequence of applying the Eddington-Finkelstein coordinates will play a key role in study of moving quark in fluid medium in next sections.

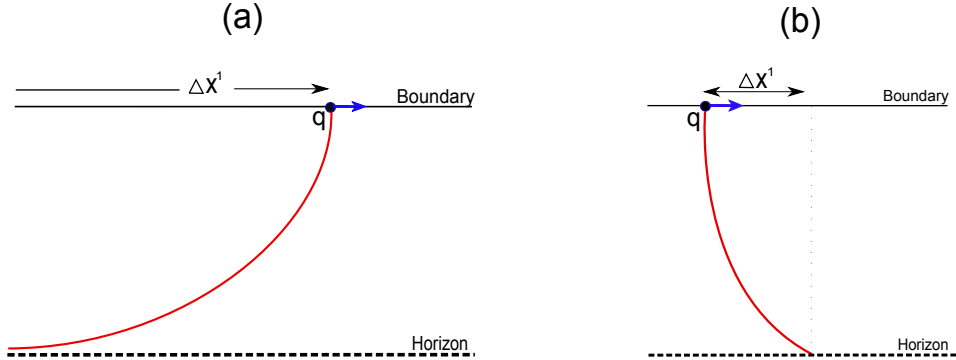


Figure 1: Comparison of the string embedding in the bulk between two different coordinate systems. In the Eddington-Finkelstein coordinates, string has been stretched in front of the moving quark in a finite range on the boundary.

### 3.3 Covariant results

In this subsection we want to present our previous result in the frame-independent expressions. It should be noticed that we have only two independent four-vectors in our analysis, four-velocity of the quark and of the plasma. So to have a covariant drag force we construct a linear combination of these velocities and demand the perpendicularity condition in (2.4). We obtain

$$F^\mu = \frac{1}{2\pi\alpha'} \frac{1}{b^2} ((u_q \cdot u_p) u_q^\mu + u_p^\mu) \quad (3.14)$$

where  $u_q^\mu$  and  $u_p^\mu$  are the four-velocity of the quark and plasma respectively. The constant factor has been fixed by going to the LF where  $u_q^\mu = \gamma_q(1, u_q, 0, 0)$ ,  $u_p^\mu = (1, 0, 0, 0)$ ,  $F^\mu = \gamma_q(\frac{dE}{dt}, \frac{dp}{dt}, 0, 0)$ , and applying the LF results given in (1.1).

The position of the world-sheet horizon,  $r^*$ , as a lorentz scalar can be given in following covariant expression:

$$r^* = \frac{1}{b} \sqrt{-u_q \cdot u_p}. \quad (3.15)$$

## 4 Gravity dual to a quark in fluid

Our final goal in two next sections is to investigate the uniform motion of a quark in a conformal fluid described by the hydrodynamic regime of an interacting field theory. According to the string/CFT picture, to study the quark motion in such an interacting field theory, the string should be solved in a bulk gravity background dual to the boundary fluid flow with appropriate boundary conditions. So it is needed to recall some of the main results of fluid/gravity correspondence.

## 4.1 Review of fluid/gravity duality

Consider a 4-dimensional field theory living on the boundary of an asymptotically  $AdS_5$  space-time. It has been shown that there is a map between the hydrodynamic regime of the strongly coupled boundary field theory and a class of inhomogeneous, dynamical black hole solutions in AdS space-time [21, 22]. In fact the long wavelength perturbations of an asymptotically AdS black brane are mapped to corresponding dynamical flows of a conformal fluid on the boundary.

According to [15], the dual gravity of an arbitrary boundary flow can be found perturbatively, order by order in a boundary derivative expansion. It means that the gravity dual to a boundary fluid flow with velocity  $u^\mu(x^\alpha)$  and temperature  $T(x^\alpha)$  can be obtained perturbatively, order by order by adding appropriate correction terms to the localized version of the metric (3.1);

$$ds^2 = -2u_\mu(x^\alpha)dx^\mu dr + r^2[P_{\mu\nu}(x^\alpha) - f(b(x^\alpha)r)u_\mu(x^\alpha)u_\nu(x^\alpha)]dx^\mu dx^\nu + ds_{cor}^2, \quad (4.1)$$

where  $P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu$  is the projection tensor. The first-order correction of the metric has been given in A.2.1.

Although the above gravity set up seems so general, one may be interested to take into account forced fluid flows rather than such free flows on the boundary. This is the subject of [23] in which, the bulk solution dual to an arbitrary fluid dynamical flow of the boundary field theory with an arbitrary slowly varying coupling, living on a weakly curved manifold, has been found up to second order in boundary derivative expansion. In order to describe such dynamical flows, instead of  $S = \int \mathcal{L}$  the action of the boundary field theory is in the form  $S = \int \sqrt{g}e^{-\phi}\mathcal{L}$  where  $g_{\mu\nu}$  is the weakly curved metric of the boundary and  $\phi(x^\mu)$  is an arbitrary slowly varying function. From this action, the Navier-Stokes equations will appear with a forcing term in the right hand side:

$$\nabla_\mu T^{\mu\nu} = e^{-\phi} \mathcal{L} \nabla^\nu \phi. \quad (4.2)$$

In [23], it has been explained that the gravity dual to an arbitrary general fluid dynamical flow in this field theory is a long wavelength solution of the Einstein-dilaton system with appropriate boundary conditions. Note that for dilaton field  $\Phi(r, x^\mu)$  in the bulk, the boundary condition is the necessity of this field to be asymptoted to the given slowly varying function  $\phi(x^\mu)$  on the boundary. Requiring expected boundary conditions, both metric of the bulk and  $\Phi(x^\mu)$  can be computed perturbatively, order by order in boundary derivative expansions. This perturbative procedure adds derivative correction terms to zero-order expressions of the metric and the dilaton field in each order of perturbation:

$$ds^2 = -2u_\mu(x^\alpha)dx^\mu dr + r^2 [P_{\mu\nu}(x^\alpha) - f(b(x^\alpha)r)u_\mu(x^\alpha)u_\nu(x^\alpha)]dx^\mu dx^\nu + ds_{cor}^2$$

$$\Phi(r, x^\mu) = \phi(x^\mu) + \Phi_{cor}(r, x^\mu) \quad (4.3)$$

where  $P_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$  is the projection tensor. The first-order correction of the metric and the dilaton have been given in A.2.2.

Emphasised by the authors of [15], a key point in all above derivations is the application of ingoing Eddington-Finkelstein coordinates which in addition to a transparent presentation of horizon regularity, provide a clean picture of bulk gravity dual to the locally equilibrated fluid domains on the boundary. In this picture, each locally equilibrated domain in the boundary fluid, extends along the ingoing null geodesics in an entire tube into the bulk. The bulk space-time within each tube is approximately a uniform AdS black brane where the width size of these tubes is the scale of variations in the fluid, namely  $L$ . As we know, having a gravity perturbative solution in boundary derivative expansion is possible, provided

$$L T \gg 1 \quad (4.4)$$

at each point on the boundary.  $T$  is the local temperature at the point.

Fluid/gravity duality has a specific characteristic. In each order of perturbation the relevant gravity solution is dual to the fluid dynamical flow in one lower order. Nevertheless, the stress tensor of the fluid will be determined at the same order to which the metric has been solved. Let us give an example. The first-order gravity solution is dual to an ideal fluid flow (zero order in hydrodynamic expansion) on the boundary but it provides the viscous fluid energy momentum tensor (first-order in hydrodynamic expansion). [15].

## 4.2 Construction of perturbative procedure

After the above brief review of fluid/gravity duality, we are going to study the quark motion in a fluid background. Going from global equilibration state of the plasma to the hydrodynamic regime, the physical quantities related to the quark motion get hydrodynamic corrections. Meanwhile, the dual string picture of the quark changes accordingly. The main idea of our paper is that for a wide range of quark velocities, one can compute these corrections perturbatively, order by order in a boundary derivative expansion.

The key point that allow us to use hydrodynamic perturbation procedure is exploiting the Eddington-Finkelstein coordinates by which, the whole string extension in the bulk may lie within just one tube. If such an extension exists, we can use the *ultralocal* method described in [15] to find the string solution perturbatively. To do that, we start with a string extended in the bulk and restricted to a tube with size  $1/T$ . At zero order of perturbation, this assumption allows us to neglect the velocity and temperature variations on the the boundary slice of the tube, namely the boundary patch. Simply speaking, the difference in values of hydrodynamic quantities between two different points in a patch, is in the next order of perturbation.

So far, we have described a string embedded in a uniform AdS black brane space-time and therefore the drag force can be computed just like as was done in § 3 where, the velocity of plasma can be chosen equal to the fluid velocity in an arbitrary point in the boundary patch i.e. in the position of the quark. So, as far as we can neglect the variation of the hydrodynamic quantities in a patch, the drag force exerted on a quark in a fluid can be computed by limiting (3.14) where the fluid global quantities are replaced by the local ones in the quark position. Consequently, we have

$$F^\mu(x^\alpha) = \frac{1}{2\pi\alpha'} \frac{1}{b^2(x^\alpha)} [(u_q \cdot u_f(x^\alpha))u_q^\mu + u_f^\mu(x^\alpha)] \quad (4.5)$$

where  $u_f^\mu$  is the fluid four-velocity and  $x^\alpha$  are the coordinates of the quark position. Since the local profile of the fluid has entered, the above expression identifies the drag force at ideal order. It is exactly similar to identification of the ideal fluid stress tensor in [15].

Computing the drag force at the level of viscous fluid needs to considering derivative corrections. Exploiting our perturbative procedure, we proceed to find the string solution in first-order corrected gravity background as the dual of a quark motion in a viscous fluid. It should be noted that our constraint of the string stay in a single tube is highly restrictive. This strict condition emphasises that to compute the hydrodynamic corrections to string embedding, one should choose the rest frame of the quark (RF) where this restriction has a manifest physical meaning and can be easily implemented in.

Now, we want to explore that under which circumstances in boundary side, the whole string extension stays in a single tube? Consider a one directional fluid dynamical flow in the LF. At zero order, where the velocity and temperature are global in a patch, the extension of the string in 1-direction of the boundary is given by (3.13) and in the RF in the fluid we have

$$\Delta x^1 = \frac{\pi}{4} b(x_0^\alpha) u^1(x_0^\alpha) = \frac{1}{4} \frac{1}{T(x_0^\alpha)} u^1(x_0^\alpha) \quad (4.6)$$

where  $u^1 = \frac{\beta}{\sqrt{1-\beta^2}}$  and  $x_0^\alpha$  is the position of the quark.

It is clear from the above discussion that as far as the string extension is of order  $1/T$ , the string lies in a tube. However (4.6) shows that the restriction of the string to stay in a single tube, limits the relative quark-fluid velocity to values lower than an upper bound. Demanding  $|\Delta x^1| < 1/T$ , one can find

$$\beta < \frac{4}{\sqrt{17}} \simeq 0.97. \quad (4.7)$$

This upper bound for the fluid velocity in translates to the upper bound of quark velocity in LAB

$$v_{u.b.} \sim 0.97 \quad (4.8)$$

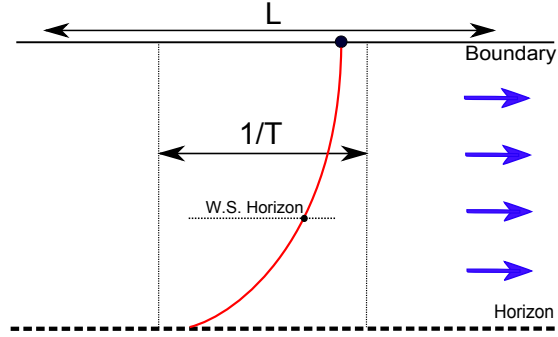


Figure 2: String in a bulk tube. In the RF and in the Eddington-finkelstein coordinates, the trail of the string is in opposite direction of the flow.

and Since it turns out to be very close to the velocity of light, one need not worry about applying our perturbative method to relativistic cases. In practical cases, the charm quarks created with energies less than  $6\text{Gev}$  satisfy our criteria.

As a result, we have shown that when the relative velocity of quark and fluid is lower than (4.8), drag force can be expressed by (4.5) at ideal order. This expression is independent of fluid dynamical flow and can be applied to any arbitrary flow which is compatible with fluid dynamics equations of motion. This result is in perfect agreement with [16, 17], where the computed drag force exerted on an external quark moving in the ideal Bjorken fluid is equal to the localized version of the drag force in global equilibrated plasma. But our construction shows that there is a concrete restriction on applying such local drag force: being the velocity of quark smaller than the upper bound (4.8).

What can we say about a fast quark with a velocity bigger than the above relativistic bound? In this case, the different points on the string do not lie in a boundary patch. Therefore, the drag force can not be a local function of the quark position which means that the drag force at some point in the fluid is a function of fluid dynamical fields (temperature and velocity) at some other point. This nonlocality is an AdS/CFT prediction which has not been obtained yet in the field theory side. As in [24] pointed, the part of string above the world sheet horizon in the bulk corresponds to the "gluon cloud" of quark in plasma. Going with a velocity bigger than the upper bound leads to exiting the gluon cloud from the boundary fluid patch. Quark's gluon cloud lags behind it and so the nonlocal drag force appears. This is just an intuitive interpretation of this effect in the field theory side. this point requires further investigation which we leave to future works.

To compute viscous corrections to drag force, we make attempt to find the string solution in the first-order corrected gravity by applying our perturbative procedure. It will be the subject of the two next sections.

## 5 Quark in temperature varying plasma

In order to compute derivative corrections to drag force exerted on a quark in fluid medium, we first choose a simple but instructive example. We will see that this toy model problem gives more insight into our perturbative procedure and reveals some of the ingredients discussed in § 4.2.

Let us consider a quark uniformly moving in the 1-direction of a globally equilibrated plasma. If the temperature in the LF is globally varying in time, it seems to be as a dynamical flow with the following profile:

$$u_f^\mu = (1, 0, 0, 0), \quad T(t). \quad (5.1)$$

But as it can be simply seen, this profile does not solve the free fluid equations of motion,  $\nabla_\mu T^{\mu\nu} = 0$ . Instead, applying an appropriate varying dilaton function, this profile can be accounted as a forced fluid flow on a flat background. According to [23], the forced Navier-Stokes equations, (4.2), at the level of leading contribution of the dilaton function is given by:

$$\nabla_\mu T^{\mu\nu} = -(\pi T)^3 \partial^\nu \phi \partial_0 \phi \quad (5.2)$$

where, at ideal order, the dilaton field does not give any contribution and therefore the temperature remains constant in time. So, the given profile in (5.1) changes to:

$$u_f^\mu = (1, 0, 0, 0), \quad T = \text{Const.}, \quad \phi(t) \quad (5.3)$$

which says that at ideal order, a rest fluid with varying temperature does not exist. Instead, we could have a thermal field theory with a slowly time varying dilaton function. From the previous section, we recall that the drag force at ideal order can be obtained by localizing the thermodynamic drag force. Restoring dilaton factor in the computations of § 3.1, we can rewrite the rate of momentum loss in the fluid frame as

$$f^1 = \frac{dp}{dt} = \frac{\pi}{2\alpha'} e^{\frac{1}{2}\phi(t)} T^2 \frac{u_q}{\sqrt{1 - u_q^2}} \quad (5.4)$$

where from [23],  $\Phi(t)$  is equal to  $\phi(t)$  at zero order of gravity set up. From the string theory we remember that in the presence of a nontrivial dilaton field, the string coupling would be changed by a dilaton factor and so the 'tHooft coupling [1]

$$\lambda = g_{YM}^2 N_c = 4\pi g_s N_c = \frac{1}{\alpha'^2} \quad (5.5)$$

would be changed too. We see that requesting to have a varying temperature plasma is

equivalent to varying the coupling in this medium. Consequently equation (5.4) gives drag force exerted on a moving quark in a plasma whose coupling is slowly varying in time.

Now, let us consider a quark in this medium and investigate the first-derivative corrections, namely viscous corrections, to drag force. According to the previous section discussion, an appropriate frame to study the derivative corrections is the RF, so hereafter we assume the slowly time variation of dilaton to be occurred at this frame. At viscous order, the profile of such flow is given by [23]

$$u_f^\mu = (u^0, u^1, 0, 0), \quad T(\tau), \quad \phi(\tau) \quad (5.6)$$

where  $u^1 = -\gamma u_q$  and  $\tau$  is the time coordinate in the RF. Based on [23], the dual picture of such flow in gravity side is an Einstein-dilaton system whose metric does not get correction up to the first order and first-order correction to the dilaton field is given by

$$\Phi_{cor}(x^\mu, r) = b \partial_\tau \phi \int_{rb}^\infty dr \frac{r^3 - 1}{r^5 f(r)}. \quad (5.7)$$

As before in the static gauge  $(\sigma, \tau) = (r, \tau)$ , the embedding of the string may be written as  $X^\mu(r, \tau) = (r, \tau, X^i(r, \tau))$ . Taking the following ansatz

$$X^1 = \xi(r, \tau), \quad X^2 = X^3 = 0, \quad (5.8)$$

the Nambu-Goto lagrangian will be given by

$$\mathcal{L} = e^{\frac{1}{2}\Phi(r, \tau)} \sqrt{A\xi'^2 + 2B\xi' + C}. \quad (5.9)$$

In the above expression,  $A = A_1$ ,  $B = B_1 + B_2\dot{\xi}$ ,  $C = C_1 + 2C_2\dot{\xi} + C_3\dot{\xi}^2$ , where among the subscripted coefficients only  $A_1$  is a function of time and the others are constant (A.3). Also  $\xi'$  and  $\dot{\xi}$  are partial derivatives of  $\xi$  with respect to  $r$  and  $\tau$ .

As discussed in § 3.1 we should find the world-sheet currents to compute the drag force. Based on our discussion in § 2, drag force in the present case is given by (3.8) similarly. From (5.9), it's clear that  $\Pi_M^\alpha$ s are not all conserved and we have

$$\partial_\alpha \Pi_{1,2,3}^\alpha = 0, \quad \partial_\alpha \Pi_\tau^\alpha = f_\tau. \quad (5.10)$$

Note that  $\alpha = r, \tau$  are the world-sheet coordinates and  $\mu = \tau, 1, 2, 3$  are the boundary directions. The existence of time dependency in the action, does not allow  $\Pi_1^\tau$  to be constant along the string. Physically, this variation along the string is related to the energy and momentum transfer between the string and the non-static bulk of the space-time. Thus by integrating the equation  $\partial_\alpha \Pi_1^\alpha = 0$  over the  $r$  component of the world-sheet



we obtain:

$$\Pi_1^r(r, \tau) - \Pi_1^r(r_0, \tau) = -\frac{d}{d\tau} \int_{r_0}^r \Pi_1^\tau(r', \tau) dr' \quad (5.11)$$

where

$$\Pi_1^r(r, \tau) = -\frac{1}{2\pi\alpha'} e^{\frac{1}{2}\Phi(r, \tau)} G_{1\nu} \frac{(\dot{X} \cdot X)(\dot{X}^\nu)' - (\dot{X})^2 (X^\nu)'}{\sqrt{-g}} = -\frac{1}{2\pi\alpha'} \pi(r, \tau). \quad (5.12)$$

To study the string embedding, we derive  $\xi'$  from the above equality

$$\xi' = -\frac{B}{A} \pm \frac{1}{A} \sqrt{\pi^2 e^{-\Phi(r, \tau)} \frac{B^2 - AC}{\pi^2 e^{-\Phi(r, \tau)} - A}} \quad (5.13)$$

with  $A$ ,  $B$  and  $C$  introduced in (5.9). Although the presence of a slowly varying dilaton function have changed the numerator and denominator in comparison with (3.4), just like there we demand the second term in (5.13) to be real. Again, this reality condition obliges the numerator and denominator to have common root. To compute this root perturbatively, we add a correction term to the embedding as

$$\xi(r, \tau) = \xi_0(r) + \xi_{cor}(r, \tau). \quad (5.14)$$

$\xi_0(r)$  is the zero-order embedding given in (3.12) in which  $u_q$  vanishes in RF:

$$\xi_0(r) = u_f^1 b \left( \tan^{-1}(rb) - \frac{\pi}{2} \right). \quad (5.15)$$

As a remarkable point, the above expression is independent of  $\Phi$ ; something that can be simply traced in § 3. The other point is the constancy of  $b$  at this order. It is a direct consequence of equation (5.2). Inserting (5.14) in (5.13) and Solving  $B^2 - AC = 0$  up to first order, we obtain the first-order corrected position of the world-sheet horizon  $r^*(\tau) = r_0^* = \frac{1}{b} \sqrt{u_f^0}$ . We see that the presence of such a dilaton function has not changed  $r^*$  up to the first order, Confirming our expectation that a scalar field does not influences the geometry at the order. In order for  $r^*$  to be also the root of the denominator up to the first order, we should have:

$$\begin{aligned} \pi(r^*, \tau) &= -e^{\frac{1}{2}\Phi(r^*, \tau)} \frac{u_f^1}{b^2} \\ &= -\left( 1 + \frac{1}{2} b u_f^0 \partial_\tau \phi \int_{r_0^* b}^\infty dr \frac{r^3 - 1}{r^5 f(r)} \right) e^{\frac{1}{2}\phi(\tau)} \frac{u_f^1}{b^2} \end{aligned} \quad (5.16)$$

As clearly observed, the only viscous contribution to above expression is related to the

first-order correction of  $\Phi$  given in (5.7).

Up to now, we have succeeded to compute the value of  $\Pi_1^r(r, \tau)$  at a special point on the string. In fact this point is the first-order corrected horizon of the world-sheet which helps us to compute  $\Pi_1^r(0, \tau) |_{r \rightarrow \infty}$  from (5.11). To compute the right hand side of (5.11) up to the first order, the integrand should be worked out at zero order:

$$\Pi_1^r(r, \tau) = -\frac{1}{2\pi\alpha'} e^{\frac{1}{2}\Phi(r, \tau)} G_{1\nu} \frac{(\dot{X} \cdot X')(X^\nu)' - (X')^2(\dot{X}^\nu)}{\sqrt{-g}} = \frac{1}{2\pi\alpha'} e^{\frac{1}{2}\Phi(r, \tau)} \frac{u_f^0 u_f^1}{1 + r^2 b^2} \quad (5.17)$$

Inserting (5.16) and (5.17) in (5.11) with choosing  $r_0 = r^*$ , we obtain  $\Pi_1^r(r, \tau)$  up to the first order as following:

$$\Pi_1^r(r, \tau) = \frac{1}{2\pi\alpha'} e^{\frac{1}{2}\phi(\tau)} \frac{u_f^1}{b^2} \left[ 1 + u_f^0 b \dot{\phi}(\tau) \left\{ \frac{1}{4} \left( \lim_{r \rightarrow \infty} g(r) - g(r_0^*) \right) - \frac{1}{2} (h(r) - h(r_0^*)) \right\} \right] \quad (5.18)$$

where

$$\begin{aligned} g(r) &= \tan^{-1}(rb) + \frac{1}{2} \log \frac{(rb)^4}{((rb)^2 + 1)(rb + 1)^2}, \\ h(r) &= \tan^{-1}(rb). \end{aligned} \quad (5.19)$$

We can explicitly see that the value of  $\Pi_1^r(r, \tau)$  at each point on the string is finite. In order to obtain the rate of momentum loss of the quark in this medium, we rewrite (3.8) but at this time up to first order:

$$F^1 = \frac{1}{\sqrt{-g}} \Pi_1^r |_{r \rightarrow \infty} = \left[ \left( \frac{1}{\sqrt{-g}} \right)_{(0)} (\Pi_1^r)_{(0)+(1)} + \left( \frac{1}{\sqrt{-g}} \right)_{(1)} (\Pi_1^r)_{(0)} \right]_{r \rightarrow \infty}. \quad (5.20)$$

where the subscripts remark the order of perturbation. Some tedious calculation shows that the second term in the brackets vanishes on the boundary. So up to viscous order, the 1-component of drag force in the RF turns out to be as

$$F^1(\tau) = \frac{1}{2\pi\alpha'} e^{\frac{1}{2}\phi(\tau)} \frac{u_f^1}{b^2(\tau)} \left[ 1 + b(\tau) u_f^0 \dot{\phi}(\tau) \left\{ \frac{1}{2} h(r_0^*) - \frac{1}{4} g(r_0^*) - \frac{\pi}{8} \right\} \right]. \quad (5.21)$$

One can simply see from  $F_{(RF)}^\mu = \gamma(f^\tau, f^1, 0, 0)$  that the leading contribution in the above expression is in complete agreement with (5.4).

In analogy with (3.14), to have a fully covariant expression, one can appropriately combine three vectorial objects  $u_q^\mu$ ,  $u_f^\mu$ ,  $\partial^\mu \phi$  and construct a general vector which in addition

to satisfying (2.4), is equal to (5.21) in the RF. Doing these stages, the result is given by

$$F^\mu(s_1) = \frac{1}{2\pi\alpha'} \frac{e^{\frac{1}{2}\phi(s_1)}}{b^2(s_1)} \left[ 1 + u_f \cdot \partial\phi \, b(s_1) \left\{ \frac{1}{2}h(s_2) - \frac{1}{4}g(s_2) - \frac{\pi}{8} \right\} \right] ((u_q \cdot u_f)u_q^\mu + u_f^\mu) \quad (5.22)$$

where  $s_1 = x \cdot u_q$  and  $s_2 = \sqrt{-u_q \cdot u_f}$  are lorentz scalars. Scalar nature of the dilaton says that demanding to have a plasma with slowly varying coupling in the RF, induces a space-time varying dilaton in the LF. The argument of dilaton field in the latter case can be determined by the lorentz transformation of the argument in the former case:

$$RF : \quad \phi(\tau) \quad \rightarrow \quad LF : \quad \phi(\gamma(t - u_q x)). \quad (5.23)$$

Such a situation seems somewhat unreal. As a well defined problem, one may expect to be encountered with a plasma medium which its coupling varies in time only. We have also answered this problem! One can simply observe that in the non-relativistic limit,  $\phi$  in the LF behaves just like in the RF as a function of time. Therefore we can say that drag force exerted on a non-relativistic heavy quark moving in a rest viscous fluid, whose coupling is varying in time, is given by:

$$F^1(t) = -\frac{1}{2\pi\alpha'} e^{\frac{1}{2}\phi(t)} \frac{u_q}{b^2(t)} \left[ 1 + \frac{1}{16} \left( \frac{\pi}{2} - 3 \log 2 \right) \dot{\phi}(t) b(t) \right] \quad (5.24)$$

In summary, invoking the perturbative procedure explained in § 4, we computed the viscous correction to drag force exerted on a non-relativistic quark in a plasma medium with a slowly time varying coupling. As it is expectable, by an increase in coupling of the plasma, the drag force will increase.

## 6 Quark in Bjorken flow

Bjorken flow [25] has been known as a fine phenomenological description of hydrodynamic regime in heavy-ion collisions in last three decades. According to Bjorken's proposal, the post-thermalization fluid dynamics in the central rapidity region of heavy-ion collisions is longitudinal boost invariant. This symmetry fixes the velocity of the fluid automatically. Solving the relativistic hydrodynamic equations of motion determines the fully profile. In addition, it is usually assumed that there is no dependence on transverse coordinates. This assumption is in correspondence with the large nuclei limit.

Relevant coordinates to study Bjorken flow are the *proper time – rapidity* coordinates by which the Minkowski metric is be given by

$$ds^2 = -d\tau^2 + \tau^2 d\eta^2 + dx_1^2 + dx_2^2 \quad (6.1)$$

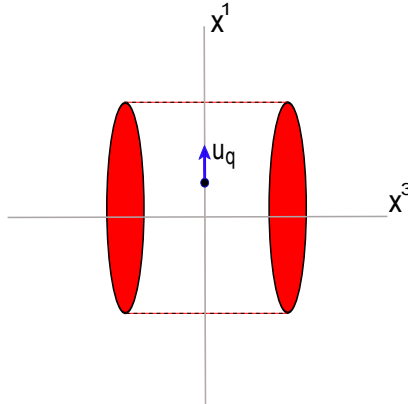


Figure 3: A transverse quark in longitudinal expanding plasma

where  $\tau$  and  $\eta$  are proper time and space-time rapidity respectively

$$\tau = \sqrt{t^2 - x_3^2}, \quad \eta = \frac{1}{2} \log \frac{t + x_3}{t - x_3}. \quad (6.2)$$

The ideal Bjorken flow in this coordinates can be described by

$$u^\mu = (1, 0, 0, 0), \quad T(\tau) \sim \frac{1}{\tau^{1/3}}. \quad (6.3)$$

### 6.1 Perturbative computations

At first time, the gravity dual of Bjorken flow was constructed by JP in the context of AdS/CFT. In this derivation, the bulk metric has been computed for asymptotic states, namely large times, of the flow by demanding the Einstein equations to have nonsingular solution [14]. Studying the energy and momentum loss of a transverse quark in this flow is accounted as a phenomenological problem. This issue has been addressed in [16, 17] where the authors have similarly applied the JP gravity set-up to compute the drag force at ideal order. We choose a different approach to compute the drag force up to viscous order. In this way, we exploit the proper time-rapidity coordinates in which, Bjorken flow is at rest. As depicted in Fig.3, we restrict the computation to the case of a quark with zero rapidity. For such a moving quark we have

$$\eta = 0, \quad \tau = t. \quad (6.4)$$

In order to use our perturbative procedure in § 4, we make a transverse boost to the RF. The quark encounters with a global boosted plasma in this frame. Symmetry considerations allow us to get the motion in the 1-direction. Consider the quark is moving with velocity

$u_q$  in the LF. So in the RF, the Bjorken flow is given by the following profile

$$u_f^\mu = (u^0, u^1, 0, 0), \quad T(u^0 \tilde{\tau} - u^1 \tilde{x}^1) \quad (6.5)$$

where tilded coordinates are related to the RF and  $u^1 = -\gamma_q u_q$ .

The gravity dual of the above flow can be derived from (4.1) as following

$$\begin{aligned} ds^2 = & -2u_0 d\tilde{\tau} dr - 2u_1 d\tilde{x}^1 dr + r^2 (d\tilde{x}^2)^2 + r^2 \tau^2 d\tilde{\eta}^2 \\ & + r^2 \left( \frac{u_0^2}{b^4 r^4} - 1 + C_{\tau\tau} \right) d\tilde{\tau}^2 + r^2 \left( \frac{u_0^2 u_1}{b^4 r^4} + C_{\tau 1} \right) d\tilde{x}^1 d\tilde{\tau} + r^2 \left( \frac{u_1^2}{b^4 r^4} + 1 + C_{11} \right) (d\tilde{x}^1)^2 \end{aligned} \quad (6.6)$$

where the derivative corrections are given by

$$\begin{aligned} C_{\tau\tau} &= \frac{2}{3r} \frac{b r F(br) u_1^2 - u_0^2}{u_0 \tilde{\tau} + u_1 \tilde{x}^1} \\ C_{\tau 1} &= \frac{2}{3r} \frac{u_0 u_1 (b r F(br) - 1)}{u_0 \tilde{\tau} + u_1 \tilde{x}^1} \\ C_{11} &= \frac{2}{3r} \frac{b r F(br) u_0^2 - u_1^2}{u_0 \tilde{\tau} + u_1 \tilde{x}^1} \end{aligned} \quad (6.7)$$

In the above expressions, for brevity, we have written  $b$  for  $b(u^0 \tilde{\tau} - u^1 \tilde{x}^1)$ .

In order to construct the string solution in the above gravity background we choose the static gauge  $(\sigma, \tau) = (r, \tilde{\tau})$ . We assume that

$$X^1 = \xi(r, \tilde{\tau}), \quad X^2 = X^3 = 0 \quad (6.8)$$

where  $X^M(r, \tilde{\tau})$  specifies the embedding of the string in the space-time. The Nambu-Goto lagrangian is given by (3.3) and  $A, B, C$  are given in A.4. Also  $\xi'$  and  $\dot{\xi}$  are partial derivatives of  $\xi$  with respect to  $r$  and  $\tilde{\tau}$ .

In here there are some noticeable differences in the drag force computation in comparison within § 5. Firstly, the zero-order embedding of the string is a function of both  $\tilde{x}^1$  and  $\tilde{\tau}$  through dependency on  $b$ . Secondly, since the translational invariance of the action has been broken in all boundary directions, the equations of motion in these directions may be written as follows

$$\partial_\alpha \Pi_{\tilde{\mu}}^\alpha = f_{\tilde{\mu}}, \quad f_{\tilde{\mu}} = \frac{\partial \mathcal{L}}{\partial x^{\tilde{\mu}}} \quad (6.9)$$

where  $\tilde{\mu}$  are the boundary directions in the RF. As our previous examples, we need to evaluate  $\Pi_1^r$  at the boundary to compute the drag force. So again, we use (3.4) and (3.9) by a small difference. We exchange  $\pi_\xi$  with  $\pi(r, \tilde{\tau})$  in this case. We recall from § 3 that

at zero order,  $B^2 - AC$  vanishes somewhere in the bulk, namely the world-sheet horizon. Requiring the string to be stretched from the boundary to the horizon implies that the denominator should be also vanished at this point. The position of this point will be corrected in each order of perturbation. To find the first-order correction, consider

$$\xi(r, \tilde{\tau}) = \xi_0(r, \tilde{\tau}) + \xi_{cor}(r, \tilde{\tau}) \quad (6.10)$$

where

$$\xi_0(r, \tilde{\tau}) = u^1 b(u^0 \tilde{\tau}) \left[ \tan^{-1}(r b(u^0 \tilde{\tau})) - \frac{\pi}{2} \right] \quad (6.11)$$

is the zero-order embedding which has been constructed by localizing  $b$  in the thermodynamic embedding in (3.12). Inserting (6.10) in (3.4), and solving  $B^2 - AC = 0$ , we obtain the first-order corrected position of the world-sheet horizon as  $r^*(\tilde{\tau}) = r_0^*(\tilde{\tau}) + r_1^*(\tilde{\tau})$  where

$$\begin{aligned} r_0^*(\tilde{\tau}) &= \frac{\sqrt{-u_0}}{b} \\ r_1^*(\tilde{\tau}) &= \frac{u_0 u_1}{2 b^4 r_0^{*3}} \dot{\xi}_0(r_0^*, u^0 \tilde{\tau}) - \frac{u_0^2}{b^5 r_0^{*3}} \xi_0(r_0^*, u^0 \tilde{\tau}) b'(u^0 \tilde{\tau}) \\ &\quad + \frac{1}{6} \left[ 2u_1^2 r_0^* F(br_0^*) + u_0^2 \frac{1}{b} \left( \frac{F(br_0^*)}{r_0^{*3} b^3} - r_0^* b F(br_0^*) - 1 \right) \right] \frac{b}{u_0 \tilde{\tau}} \end{aligned} \quad (6.12)$$

where the argument of  $b$  is  $u^0 \tilde{\tau}$  in these expressions. As clearly observed in (6.12),  $r_0^*(\tilde{\tau})$  is the localized version of (3.5), reconfirmation of our discussion in § 4. It should be noticed that the derivative terms have been computed at  $\tilde{x}^1 = 0$ . Demanding  $\pi^2(r, \tilde{\tau}) - A$  to be vanished at this corrected  $r^*$ , the first-order correction to  $\Pi_1^r(r, \tilde{\tau})$  at this point can be given by

$$\begin{aligned} \Pi_1^r(r^*, \tilde{\tau}) &= \frac{1}{2\pi\alpha'} \frac{u_1}{b^2} \left[ 1 + u_0 \dot{b} \left( \tan^{-1}(r_0^* b) + \frac{r_0^* b}{1 + r_0^{*2} b^2} - \frac{\pi}{2} \right) \right. \\ &\quad \left. - 2u_1 b' \left( \tan^{-1}(r_0^* b) - \frac{\pi}{2} \right) + \frac{1}{3} \frac{b}{\tilde{\tau}} \left( F(br) - \frac{u_0^2 (2u_0^2 - 1) \sqrt{-u_0}}{u_0^2 - 1} \right) \right]. \end{aligned} \quad (6.13)$$

Having  $\Pi_1^r(r^*, \tilde{\tau})$ , one can integrate the 1-component of (6.9) over the  $r$  to derive the first-order corrected  $\Pi_1^r(r, \tilde{\tau})$  function. After some calculation we find

$$\begin{aligned} \Pi_1^r(r, \tilde{\tau}) &= \Pi_1^r(r^*, \tilde{\tau}) + \int_{r^*}^r (f_1 - \partial_{\tilde{\tau}} \Pi_1^{\tilde{\tau}}) dr \\ &= \Pi_1^r(r^*, \tilde{\tau}) - \frac{1}{2\pi\alpha'} \frac{u_1}{b^2} \left[ \tan^{-1} r b (u_1 b' + u_0 \dot{b}) + \frac{r b}{1 + r^2 b^2} (u_1 b' - u_0 \dot{b}) \right]_{r^*}^r. \end{aligned} \quad (6.14)$$

which is finite at all points on the string. In order to compute the drag force at viscous

order, we again utilize (5.20) and obtain

$$F_{RF}^1 = \frac{1}{2\pi\alpha'} \frac{u_1}{b^2} \left[ 1 + \mathcal{A} u_0 \dot{b} + \mathcal{B} u_1 b' + \mathcal{C} \frac{b}{\tilde{\tau}} \right] \quad (6.15)$$

where

$$\begin{aligned} \mathcal{A} &= 2 \tan^{-1}(r_0^* b) - \pi, \\ \mathcal{B} &= \tan^{-1}(r_0^* b) + \frac{\pi}{2} + \frac{r_0^* b}{1 + r_0^{*2} b^2}, \\ \mathcal{C} &= \frac{1}{3} \{ u_0^2 \sqrt{-u_0} + F(b r_0^*)(u_0 + u_0^3) \} \end{aligned} \quad (6.16)$$

Although the finiteness of corrections is a sign of truth for our perturbative procedure, one can separately check it by computing the first-order corrected  $\xi'$ . After tedious calculations, it will be turned out that the restriction condition on the bulk embedding of the string is satisfied and just, the upper bound of the quark velocity is corrected by a first-order contribution.

So far, we have computed the drag force in the RF. Because of breaking the rotational symmetry, we can not covariantize the drag force. So in order to find the drag expression in the LF, we first make the inverse boost as following

$$\begin{aligned} \text{four-velocity of the quark : } (1, 0, 0, 0) &\longrightarrow (\gamma, \gamma u_q, 0, 0) \\ \text{quark coordinates : } (\tilde{\tau}, 0, 0, 0) &\longrightarrow \left( \frac{1}{\gamma} t, u_q t, 0, 0 \right). \end{aligned} \quad (6.17)$$

It should be reminded that for a quark with zero rapidity, the proper time and the LF time coordinate are coincident. The 1-component of drag force in the LF may be written as

$$F_{LF}^1 = \frac{\partial x^1}{\partial \tilde{x}^1} F_{RF}^1 + \frac{\partial x^1}{\partial \tilde{\tau}} F_{RF}^0. \quad (6.18)$$

For the sake of computing the  $F_{RF}^0$ , we need to find the  $\Pi_\tau^r$ . After some calculation similar to that for obtaining the  $\Pi_1^r$ , we get that

$$F_{RF}^0 = 0 \quad (6.19)$$

up to viscous order. As we have pointed out in § 2, this condition would be maintained even when the derivative corrections are taken into account. Physically, this means that there should not be any energy loss in the RF at viscous order, just like at ideal order. Equation (6.19) is in complete agreement with this constraint. So substituting (6.15) in (6.18) gives the rate of momentum loss in the LF. But to proceed further, it is needed to

have the relations between the derivatives in these two frames which can be written as

$$\begin{aligned} b'_{RF}(u^0 \tilde{\tau}) &= \gamma u_q \dot{b}_{LF}(t) \\ \dot{b}_{RF}(u^0 \tilde{\tau}) &= \gamma \dot{b}_{LF}(t) \end{aligned} \quad (6.20)$$

where  $u^0 = \gamma$  and  $u^1 = -\gamma u_q$ . Finally the rates of energy and momentum loss of the quark in the LF turn out to be as

$$\frac{dp}{dt} = \frac{1}{u_q} \frac{dE}{dt} = \frac{-1}{2\pi\alpha'} \frac{u_q \gamma}{b^2} \left[ 1 - \dot{b} (\gamma^2 \mathcal{A} + \gamma^2 u_q^2 \mathcal{B}) + \frac{b}{t} \gamma \mathcal{D} \right] \quad (6.21)$$

where

$$\begin{aligned} \mathcal{A} &= 2 \tan^{-1}(\sqrt{\gamma}) - \pi, \\ \mathcal{B} &= \tan^{-1}(\sqrt{\gamma}) + \frac{\pi}{2} + \frac{\sqrt{\gamma}}{1 + \gamma}, \\ \mathcal{D} &= \frac{1}{3} \left( -F(\sqrt{\gamma}) + \frac{2 + \gamma^2 u_q^2}{\gamma} + \gamma^{1/4} \right). \end{aligned} \quad (6.22)$$

We can write (6.21) in a more clear expression as following

$$\frac{dp}{dt} = \frac{1}{u_q} \frac{dE}{dt} = -\frac{\pi}{2} \sqrt{\lambda} \frac{u_q}{\sqrt{1 - u_q^2}} T^2(t) \left[ 1 + \frac{\dot{T}(t)}{\pi T^2(t)} (\gamma^2 \mathcal{A} + \gamma^2 u_q^2 \mathcal{B}) + \frac{1}{\pi T(t)t} \gamma \mathcal{D} \right] \quad (6.23)$$

According to our expectation, drag force at ideal order is the same as the drag force in thermal plasma medium but with a difference that the temperature would be localized. A remarkable point in (6.23) is the temperature profile. This is different from the ideal Bjorken flow in (6.3) and should be determined by solving the fluid dynamics equations at viscous order. This issue will be discussed in next subsection.

At the end, we should check the finiteness of the string extension in the 1-direction at viscous order. One can do this lengthy and cumbersome calculation and see that  $\xi(r)$  has finite range at viscous order, however, the finiteness of drag corrections is the result of allowability of our perturbative method.

## 6.2 Phenomenological estimates

In this subsection we want to apply our perturbative results to the phenomenology of QGP in heavy-ion collisions. As it is known, the underlying theory that describes what happens through such collisions is QCD. On the other hand, the energy scale in the QGP is related to the nonperturbative regime of QCD. Although String/CFT duality could have obtained some successes in describing of strongly coupled gauge theories, we need an appropriate



dictionary to apply the string/CFT results to QCD. In this way we use the alternative scheme introduced by Gubser in [19]. Using the static force between quarks to normalize the 't Hooft coupling, he has compared the strength of the drag force on a heavy quark moving through a thermal medium between  $\mathcal{N} = 4$  SYM gauge theory and QCD. He has chosen the energy density ( $\epsilon$ ) as a quantity for comparison. Having  $\epsilon_{SYM} = \epsilon_{QCD}$  leads to

$$T_{SYM} \approx 3^{-1/4} T_{QCD}. \quad (6.24)$$

In heavy-ion collisions, the hydrodynamic regime dominates after  $\tau_{Th} \sim 1fm$  and the expanding hot matter cools down with Bjorken profile. This goes on until the phase transition starts at  $\tau_c \sim 4fm$  with  $T_c \sim 170Mev$ . During the phase transition, the temperature is approximately constant and equal to  $T_c$ . After the phase transition is over at  $\tau_H \sim 24fm$ , the hadron fluid undergoes a hydrodynamic expansion. Finally at  $\tau_f \sim 42fm$  the hadron breakup, occurs [26]. But in  $\mathcal{N} = 4$  SYM gauge theory, there is not any phase transition. So to get estimate for QCD from this theory, we need to make some assumptions about SYM in the range of the temperatures determined by (6.24). We consider the hydrodynamic regime to be extended from  $\tau_{Th}$  to  $\tau_H$  in SYM. In addition we assume that temperature of SYM at  $\tau_H$  corresponds to  $T_{QCD} \sim 170Mev$ . Applying this assumption to (6.3), one can fix the the temperature profile at ideal order

$$T_{SYM}(\tau) = 129Mev \left( \frac{24fm}{\tau} \right)^{1/3} \quad 1fm \leq \tau \leq 24fm. \quad (6.25)$$

The temperature profile at viscous order can be derived from the fluid dynamics equations of motion by considering the first-derivative corrections. The width scale of hydrodynamic patches restricts the perturbative computations to the space-time intervals whose widths are smaller than  $1/T \sim 1fm$ . Therefore, we use the numerical method to find the temperature profile at viscous order. In order to solve the second-order equations numerically, we fix  $T_{SYM} = 170Mev$  at  $\tau = 24fm$ . The numerical results has been shown in Fig.4.

Now we are ready to use our perturbative results to obtain estimates on the contribution of the drag force on a transverse quark in expanding QGP. To proceed, we consider the drag coefficient  $\eta_D$  which is defined as follows:

$$\eta_D = -\frac{1}{p} \frac{dp}{dt} \quad (6.26)$$

and to find it in our case, we rewrite the drag force expression in (6.23) as a function of

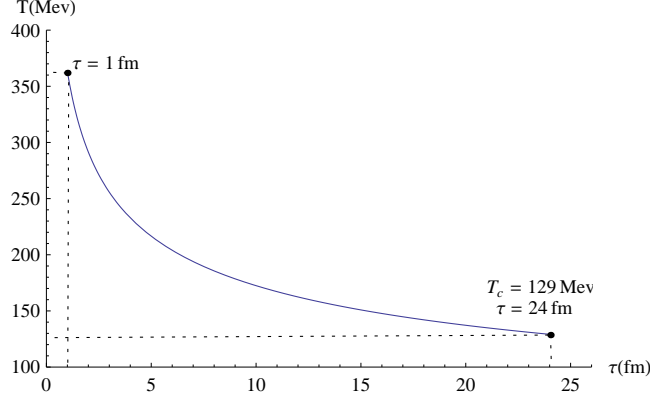


Figure 4: The numerical profile of the temperature at viscous order. This profile is so similar to the analytical profile at ideal order that the deviation could not be simply shown.

the quark momentum:

$$\frac{dp}{dt} = -\frac{\pi}{2}\sqrt{\lambda}\frac{p}{M}T^2(t)\left[1 + \frac{\dot{T}(t)}{\pi T^2(t)}\left(\frac{p^2}{M^2}\mathcal{B} + \left(1 + \frac{p^2}{M^2}\right)\mathcal{A}\right) + \frac{1}{\pi T(t)t}\left(1 + \frac{p^2}{M^2}\right)\mathcal{D}\right]. \quad (6.27)$$

$\mathcal{A}$ ,  $\mathcal{B}$  and  $\mathcal{D}$  have been introduced in (6.22) which should be expressed in terms of  $p/M$  in the above equation. In contrast to the constant drag coefficient of a quark moving in thermal plasma, the drag coefficient is a function of the quark momentum here. Although this feature appears only in viscous order, there is no a constant  $\eta_D$  even at ideal order. the time-dependency of  $T$  leads to variation of the drag coefficient.

Another important quantity is the diffusion time  $t_D$ , defined as the time it takes the momentum of the quark falls by  $1/e$ . For a charm quark ( $M=1.5$  GeV) moving through a thermal plasma with  $T_{SYM} = 190$  MeV (and equivalently  $T_{QCD} = 250$  MeV), alternative scheme gives  $t_D \approx 2.1$  fm [19]. In our case where the plasma is not in thermal equilibrium, one should solve the equation (6.27) to find the  $t_D$ . To do that, we use the numerical profile of the temperature in Fig.4 and solve the equation (6.27) numerically. In this way we need to know the initial time of creating the transverse quark ( $\tau_0$ ) and whose momentum ( $p_0$ ) at that time. Respecting our upper-band of velocity in § 4, in Fig.5 we have plotted the dependence of the diffusion time on the initial momentum for  $\tau_0 = 1$  fm case. Just like the thermal plasma medium, diffusion time of the quark is independent of the initial momentum at ideal order. The viscous curve is below the ideal line, as a direct consequence of dissipative effects in a fluid dynamical flow. Another observation in Fig.5 is that in a viscous Bjorken fluid, diffusion time decreases with increasing the initial momentum. This is a common behavior in viscous systems.

In order to complete our discussion, we now study the effect of  $\tau_0$  on the diffusion time.

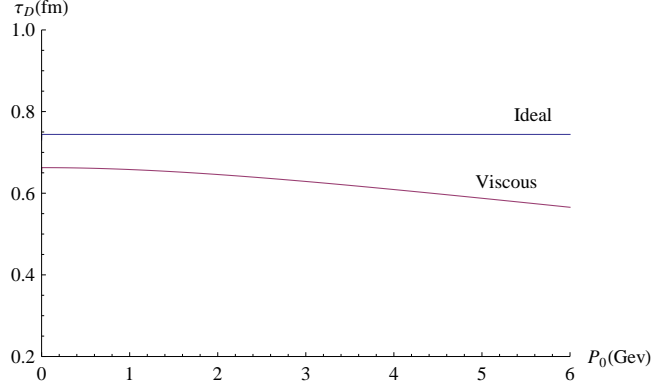


Figure 5: Comparison of the diffusion time for a transverse quark created at  $\tau = 1fm$  in Bjorken flow between ideal and viscous orders.

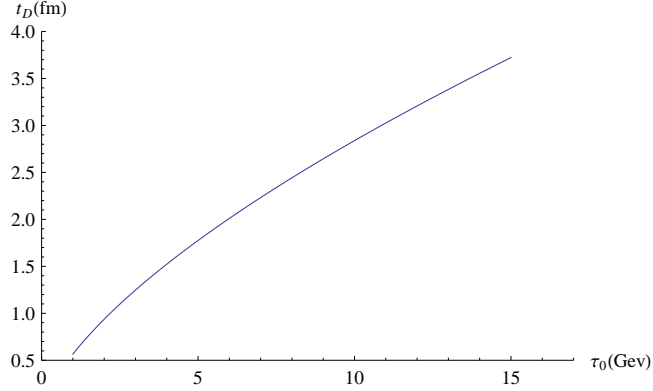


Figure 6: Diffusion time as a function of the quark creation time.  $p_0 = 6GeV$

Consider a transverse quark with a high  $p_0$  near to the upper-band of the momentum. Depending on its creation time, it will experience the cooling of the system and the change of the temperature. The later creation occurs, the smaller temperatures would be observed. The resultant variation of  $t_D$  has been shown in Fig.6. Our estimates show that based on fluid/gravity duality, the diffusion time of a high energy transverse charm quark could be about  $4fm$ , provided that it is created sufficiently late in Bjorken flow. As it can be observed, the diffusion time for a charm quark created at  $\tau_0 = 1fm$ , is about  $0.6fm$  which is similar to the result of [19] from the obvious scheme. The origin of dependency on the creation time of the quark is that the drag coefficient is time dependent in our case. In contrast to previous researches where a thermal plasma was studied,  $\eta_D$  is a function of time in a fluid dynamical flow.

## 7 Discussion

Applying AdS/CFT correspondence, many problems related to a moving quark in thermal plasma have been studied in recent years. The rate of energy and momentum loss [5–7, 7–9, 27, 28], jet quenching parameter [29], Brownian motion [30, 31], motion in a magnetic plasma [32] and etc. are all examples of a quark probe which in AdS/CFT duality is pictured to a string probe in an asymptotically AdS space-time (for brief review, see [3, 33]). In these problems, the solutions of classical string in gravity side, have been strong enough to address and clarify some questions in the strongly coupled CFT side.

It has been accepted that the expanding plasma medium generated after scattering of two nuclei evolves from a far-from-equilibrium state to the hydrodynamic regime during about 1fm. So one may be interested in studying the above problems but at this time, in a fluid dynamical background. To proceed, a string probe should be studied in a gravity space-time dual to the boundary fluid dynamical flow.

In this paper, we have shown that based on fluid/gravity correspondence, one can construct a perturbative procedure to find the string solution on a perturbed AdS background dual to a fluid dynamical flow on the boundary of AdS. According to our discussion in § 4, as far as the string probe lies in one tube in the bulk, the position of the world-sheet horizon can be determined perturbatively and so the drag force exerted on the quark will take correction, order by order in a boundary derivative expansion. Obviously, by going to higher orders in perturbation, the world-sheet horizon position and thereby the drag force get higher-derivative correction terms. Requesting the string to be always in a unique tube leads to appearance of an upper bound for the relative velocity of the quark and fluid. This upper bound is not much lower than unity and is about 0.97. As a remarkable point, the necessary condition of being string in its original tube should be checked at each order of perturbation, however the finiteness of the computed corrections is a sign of correctness in the given order of perturbation.

By this construction one can study a quark in the fluid medium for a wide range of velocities. As an important example we have computed first-derivative correction, namely viscous correction, to the drag force exerted on a transverse quark in a QGP experiment. An interesting aspect of this procedure is that the drag force at ideal order is exactly the localized version of the drag force computed in a global thermal plasma. In another words, to find drag force at ideal order, we should insert the local velocity and temperature of the fluid in the quark position into (3.14) at each instance of time.

On the other hand, existence of the velocity upper bound has a physical outcome to our problem. Consider a moving quark with a velocity even greater than the upper bound velocity given in (4.8). Destruction of our perturbative procedure is the first consequence of such motion and in addition, we conclude that even at ideal order, the drag force exerted on such quark is not a local quantity. It is reasonably related to the high velocity of the

quark. We leave further investigation on this issue to the future.

From the phenomenological point of view, our perturbative method makes improvement in previous estimates of quark diffusion time. Our estimates show that in contrast to a thermal plasma, the drag coefficient changes to be a time dependent function in a dynamical flow. Accordingly, the diffusion time will be a function of the quark creation time.

This method can open a window to some new problems. Study of rotating quark in a fluid or investigating Brownian motion of quark in a fluid dynamical flow are now possible through this method. One can generalize [32] to the context of magnetohydrodynamic perturbatively, using the metric given in [34] and applying our procedure. Motivated by [27], another instructive example is the computation of non-relativistic correction to drag force in a fluid medium which can be performed by finding the string solution in the gravity setup of [35]. As a complete generalization of our work we leave the study of a quark in a general fluid dynamical flow to our further work.

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## A Appendices

### A.1 Global boosted plasma

In this appendix we rewrite the induced metric on the world-sheet given in (4) with more detail. From (4) we have

$$g = -(A\xi'^2 + 2B\xi' + C) \quad (\text{A.1})$$

in which, the coefficients constructed from metric (3.1) are given by

$$\begin{aligned} A &= G_{t1}^2 - G_{11}G_{tt} = r^4 f(br) \\ B &= G_{tr}G_{t1} - G_{tt}G_{r1} + u_q (G_{rt}G_{11} - G_{r1}G_{t1}) = \gamma r^2 (u_q - \beta) \\ C &= (G_{rt} - u_q G_{r1})^2 = \gamma^2 (1 - \beta u_q)^2 \end{aligned} \quad (\text{A.2})$$

where  $u_q$  and  $\beta$  are the three-velocity of quark and plasma in the LF respectively.

## A.2 Fluid dynamical correction

Our perturbative computations need to have first-order corrected dual gravity. So in following, we recall the computed corrections in [15, 23] up to first order.

### A.2.1 Free fluid

The first-order correction to (4.1) has been given in equation (4.24) of [15] as follows:

$$ds_{cor}^2 = r^2 b F(b r) \sigma_{\mu\nu} dx^\mu dx^\nu + \frac{2}{3} r u_\mu u_\nu \partial_\lambda u^\lambda dx^\mu dx^\nu - r u^\lambda \partial_\lambda (u_\nu u_\mu) dx^\mu dx^\nu \quad (\text{A.3})$$

where

$$F(r) = \frac{1}{4} \left[ \ln \left( \frac{(1+r)^2(1+r^2)}{r^4} \right) - 2 \arctan(r) + \pi \right] \quad (\text{A.4})$$

and

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} \partial_{(\alpha} u_{\beta)} - \frac{1}{3} P^{\mu\nu} \partial_\alpha u^\alpha. \quad (\text{A.5})$$

### A.2.2 forced fluid

In this case, the correction to the metric has been given in equation (3.4) of [23] which is similar to the (A.3) with this difference that the derivatives are covariant derivatives. The first-order correction to the dilaton field is given by

$$\Phi_{cor} = b u \cdot \partial \phi \int_{rb}^{\infty} dx \frac{x^3 - 1}{x^5 f(x)}. \quad (\text{A.6})$$

## A.3 Varying plasma

In this appendix we rewrite all coefficient introduced in (5.9) with their expressions. Note that in following, we have applied first-order corrected metric in (4.3):

$$A_1 = G_{\tau 1}^2 - G_{11} G_{\tau\tau} = r^4 f(br), \quad (\text{A.7})$$

$$\begin{aligned} B_1 &= G_{r\tau} G_{1\tau} - G_{r1} G_{\tau\tau} = -u_f^1 r^2 \\ B_2 &= G_{r\tau} G_{11} - G_{r1} G_{\tau 1} = u_f^0 r^2 \end{aligned} \quad (\text{A.8})$$

and

$$\begin{aligned} C_1 &= G_{r\tau}^2 = (u_f^0)^2 \\ C_2 &= G_{r\tau} G_{r1} = -u_f^0 u_f^1 \\ C_3 &= G_{r1}^2 = (u_f^1)^2. \end{aligned} \quad (\text{A.9})$$

## A.4 Bjorken flow

The coefficient introduced below (6.8) have the same expressions as which given in previous appendix by this difference that inhere our  $\tau$  should be substituted by  $\tilde{\tau}$ .

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